WPM01 QUANTUM THEORY

SPH4U

CH 12 (KEY IDEAS)

- define and describe the concepts and units related to the present-day understanding of the nature of the atom
- describe the photoelectric effect in terms of the quantum energy concept
- outline evidence that supports a photon model of light
- describe and explain the Bohr model of the hydrogen atom
- collect or interpret experimental data involving the photoelectric effect and the emission spectrum of hydrogen
- outline the historical development of models of matter and energy from 1890 to 1925
- describe how the development of quantum theory has led to scientific and technological advances
- describe some Canadian contributions to modern physics

EQUATIONS

- Planck's Constant

$$h = 6.63 \times 10^{-34}\ \mathrm{J\ s}$$

- Energy of a photon

$$E_{\mathrm{photon}} = hf$$

- Einstein's Photoelectric Equation

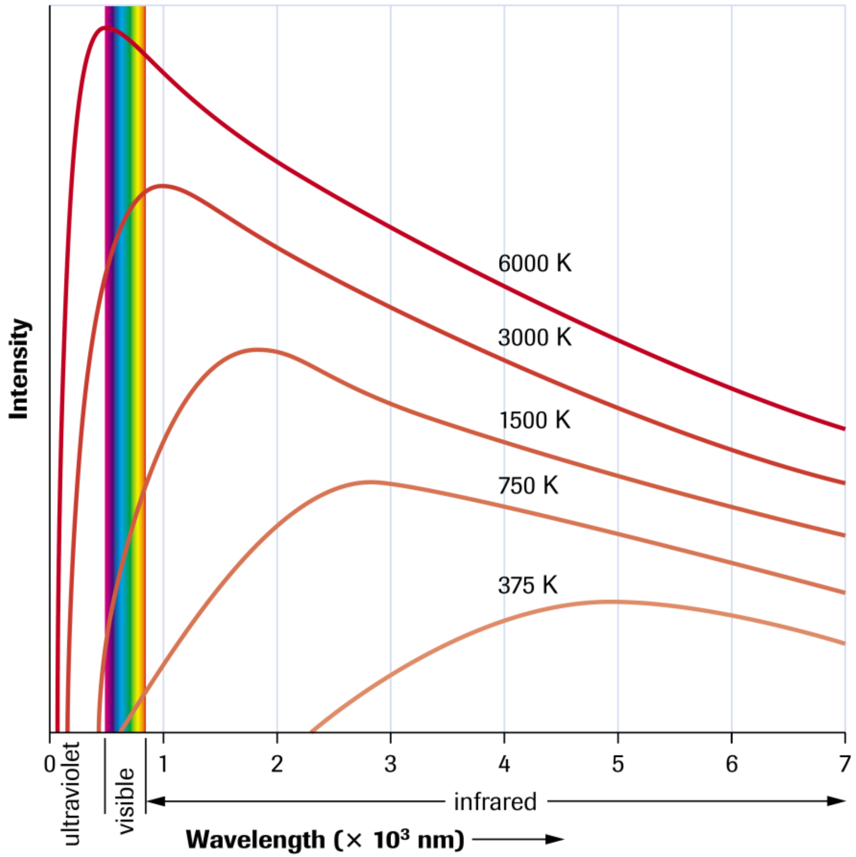
$$E_K = E_{\mathrm{photon}} - W = hf - W$$

- Momentum of a photon

$$p = \frac{\lambda}{h}$$

BLACKBODY RADIATION

- **Blackbody:** an object that completely absorbs any radiation falling upon it
- **Blackbody Radiation:** radiation that would be emitted from an ideal blackbody



PLANK'S QUANTUM HYPOTHESIS

- **Quanta:** packets of energy; one quantum is the minimum amount of energy a particle can emit
  - The energy of one quanta is proportional to the frequency of the radiation, or

$$E = hf$$

where h is a constant in joule-seconds [Js]

PLANK'S QUANTUM HYPOTHESIS – CONT.

- **Planck's constant:** constant with the value

$$\boldsymbol{h = 6.63 \times 10^{-34}\ \mathrm{J\ s}}$$

  - represents the ratio of the energy of a single quantum to its frequency

- Planck hypothesized that the emitted energy must be an integral multiple of the minimum energy, or

$$E = nhf, n = 1,2,3, \ldots$$

PROBLEM 1

Calculate the energy in joules and electron volts of

(a) a quantum of blue light with a frequency of 6.67×10^{14} Hz

(b) a quantum of red light with a wavelength of 635 nm

PROBLEM 1 – SOLUTIONS

(a) $f = 6.67 \times 10^{14}$ Hz

$h = 6.63 \times 10^{-34}$ J·s

$$E = hf$$

$$= (6.63 \times 10^{-34}\ \text{J·s})(6.67 \times 10^{14}\ \text{Hz})$$

$$E = 4.42 \times 10^{-19}\ \text{J}$$

1.60×10^{-19} J $=$ 1 eV

$$\frac{4.42 \times 10^{-19}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 2.76\ \text{eV}$$

The energy is 4.42×10^{-19} J, or 2.76 eV.

PROBLEM 1 – SOLUTIONS

(b) $\lambda = 635\text{ nm} = 6.35 \times 10^{-7}\text{ m}$

$h = 6.63 \times 10^{-34}\text{ J·s}$

$$E = hf$$

But $v = f\lambda$ or $c = f\lambda$, and $f = \frac{c}{\lambda}$, where c is the speed of light $= 3.00 \times 10^{8}\text{ m/s}$.

$$E = \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34}\text{ J·s})(3.00 \times 10^{8}\text{ m/s})}{6.35 \times 10^{-7}\text{ m}}$$

$$E = 3.13 \times 10^{-19}\text{ J}$$

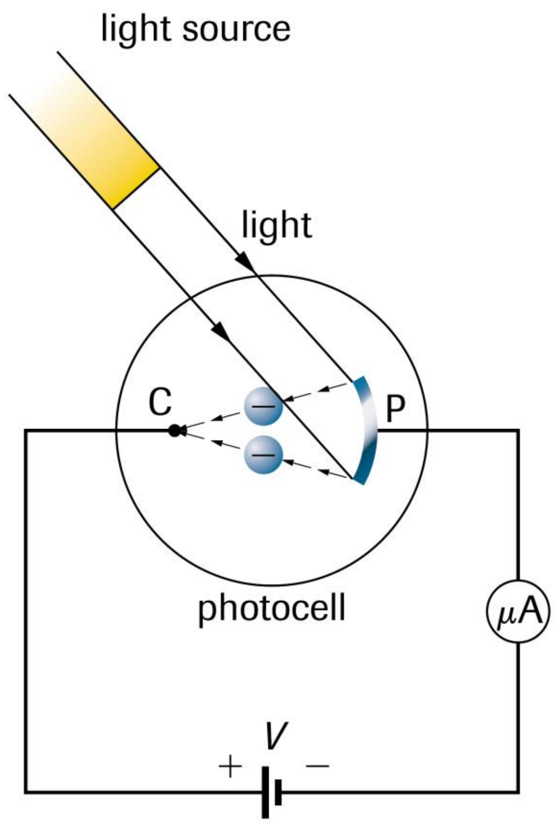
$1.60 \times 10^{-19}\text{ J} = 1\text{ eV}$

$$\frac{3.13 \times 10^{-19}\text{ J}}{1.60 \times 10^{-19}\text{ J/eV}} = 1.96\text{ eV}$$

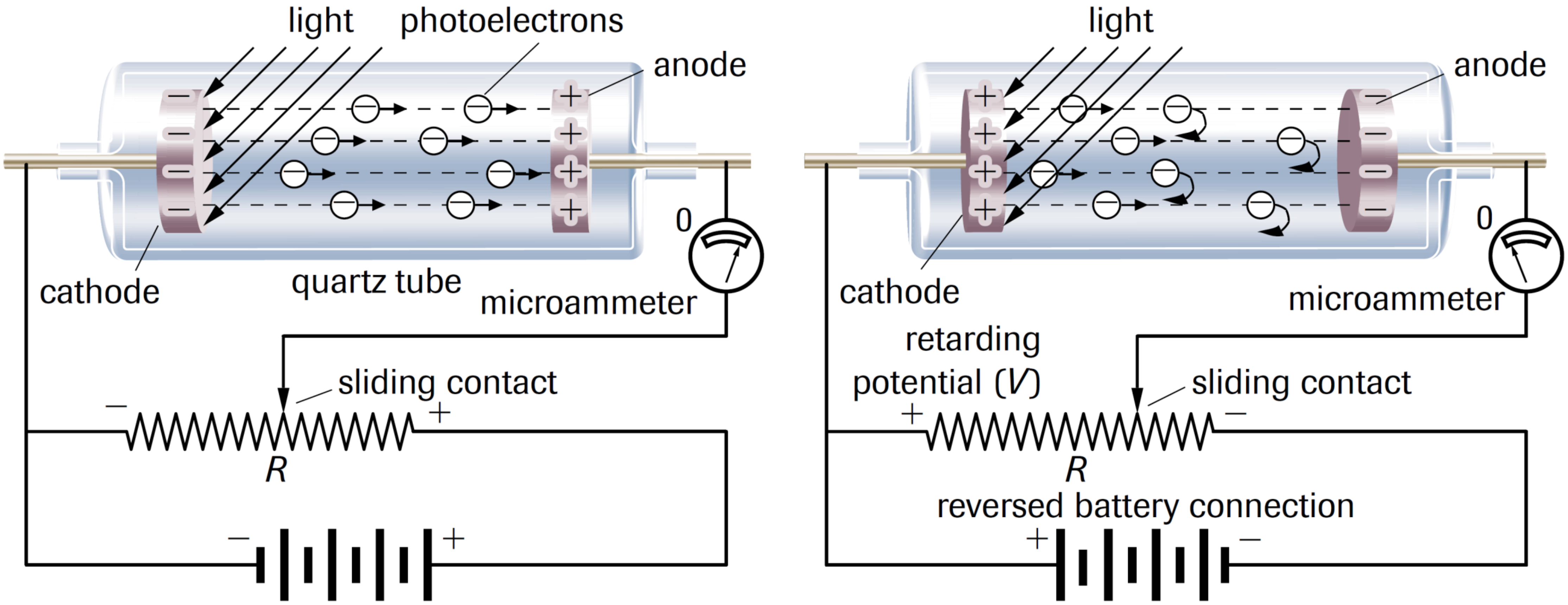
The energy is 3.13×10^{-19} J, or 1.96 eV.

EINSTEIN AND THE PHOTOELECTRIC EFFECT

- **Photoelectric Effect:** the phenomenon in which electrons are liberated from a substance exposed to electromagnetic radiation
- **Photoelectrons:** electrons liberated in the photoelectric effect
- **Cutoff Potential:** smallest potential difference sufficient to reduce the photocurrent to zero
  - Corresponds to the maximum kinetic energy of the photoelectrons

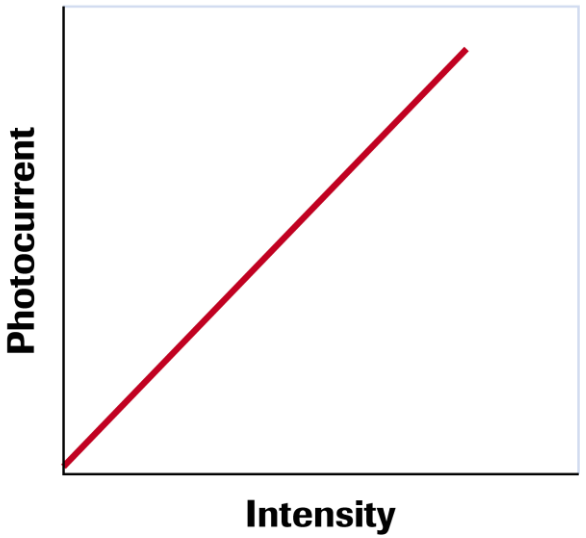


THE PHOTOELECTRIC EFFECT



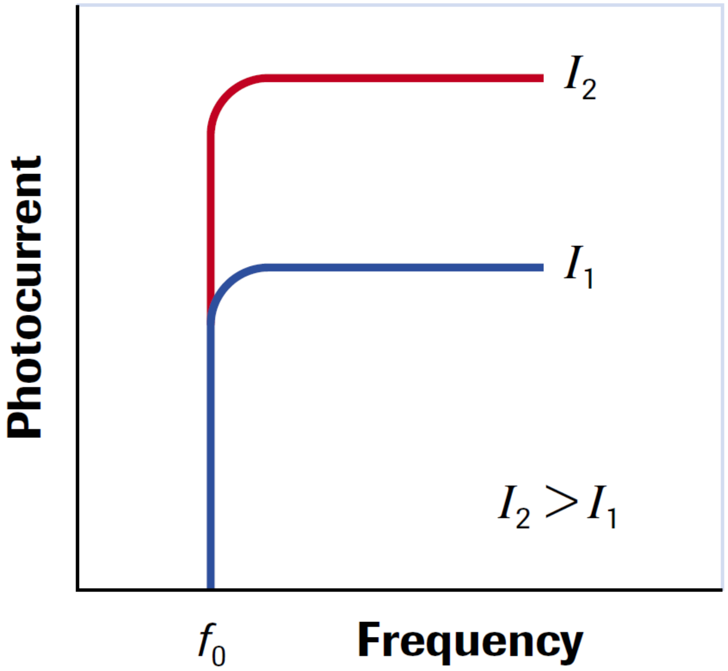
THE PHOTOELECTRIC EFFECT: SIGNIFICANT FINDINGS

1. Photoelectrons are emitted from the photoelectric surface when the incident light is above a certain frequency f_0, called the threshold frequency. Above the threshold frequency, the more intense the light, the greater the current of photoelectrons.
   - **Threshold Frequency (f_0):** the minimum frequency at which photoelectrons are liberated from a given photoelectric surface



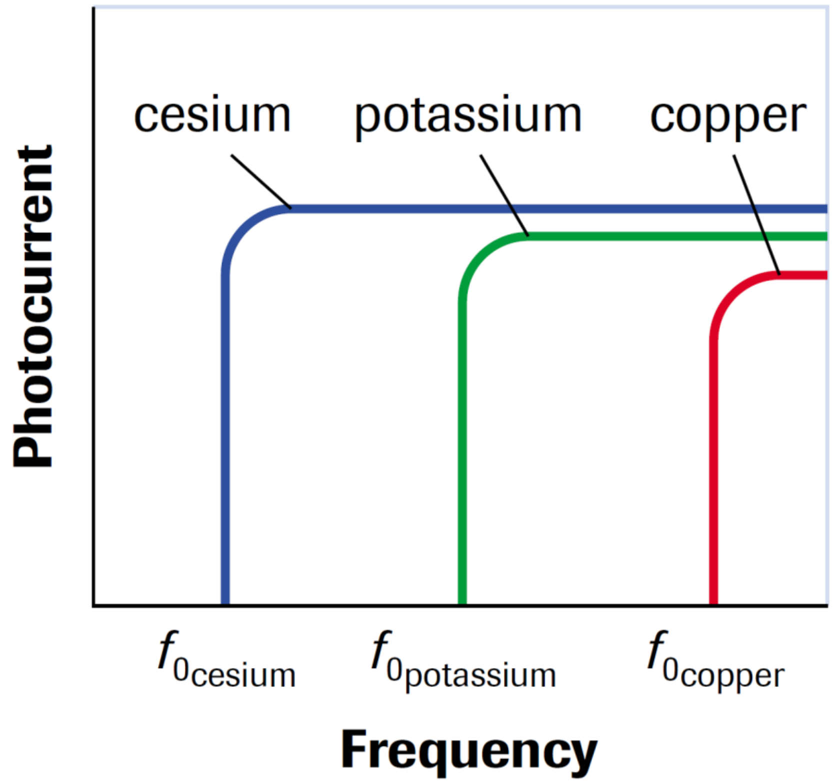
THE PHOTOELECTRIC EFFECT: SIGNIFICANT FINDINGS – CONT.

2. The intensity (brightness) of the light has no effect on the threshold frequency. No matter how intense the incident light, if it is below the threshold frequency, not a single photoelectron is emitted.



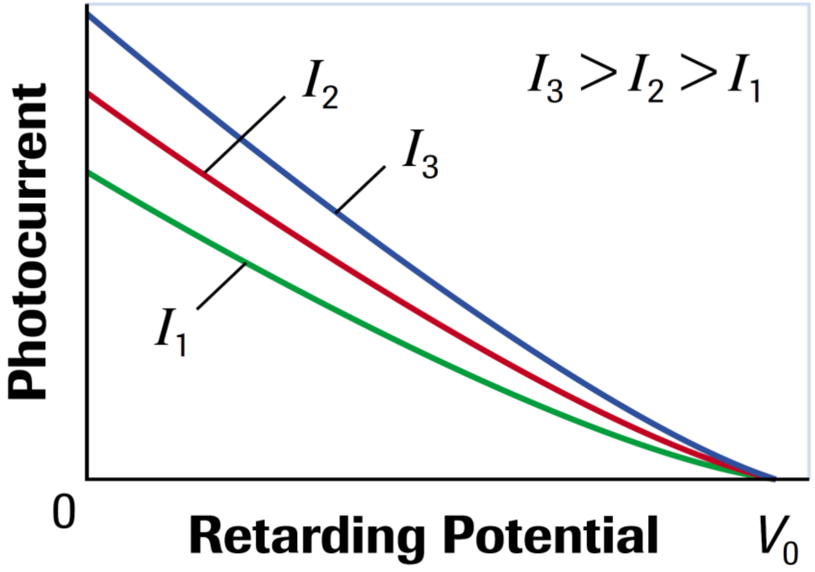
THE PHOTOELECTRIC EFFECT: SIGNIFICANT FINDINGS – CONT.

3. The threshold frequency, at which photoelectric emission first occurs, is different for different surfaces. For example, light that causes photoelectric emission from a cesium cathode has no effect on a copper cathode.



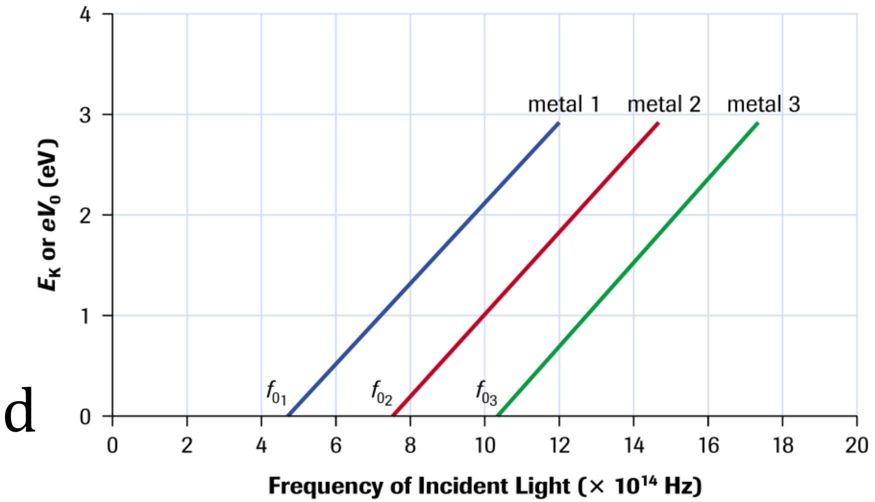
THE PHOTOELECTRIC EFFECT: SIGNIFICANT FINDINGS – CONT.

4. As the retarding potential applied to the anode is increased, the photocurrent I decreases, regardless of the intensity of the light. The photoelectrons are thus emitted with different kinetic energies. A value V_0 of the retarding potential is eventually reached, just sufficient to make the photocurrent zero. Even the fastest photoelectrons are now prevented from reaching the anode, being turned back by the retarding potential.



THE PHOTOELECTRIC EFFECT: SIGNIFICANT FINDINGS – CONT.

5. If different frequencies of light, all above the threshold frequency, are directed at the same photoelectric surface, the cutoff potential is different for each. It is found that the higher the frequency of the light, the higher the cutoff potential. The cutoff potential is related to the maximum kinetic energy with which photoelectrons are emitted: for a photoelectron of charge e and kinetic energy E_K, cut off by a retarding potential V_0, $E_K = eV_0$.
   - By illuminating several photoelectric surfaces with light of various frequencies and measuring the cutoff potential obtained for each surface.
   - Although each surface has a different threshold frequency, each line has the same slope.



THE PHOTOELECTRIC EFFECT: SIGNIFICANT FINDINGS – CONT.

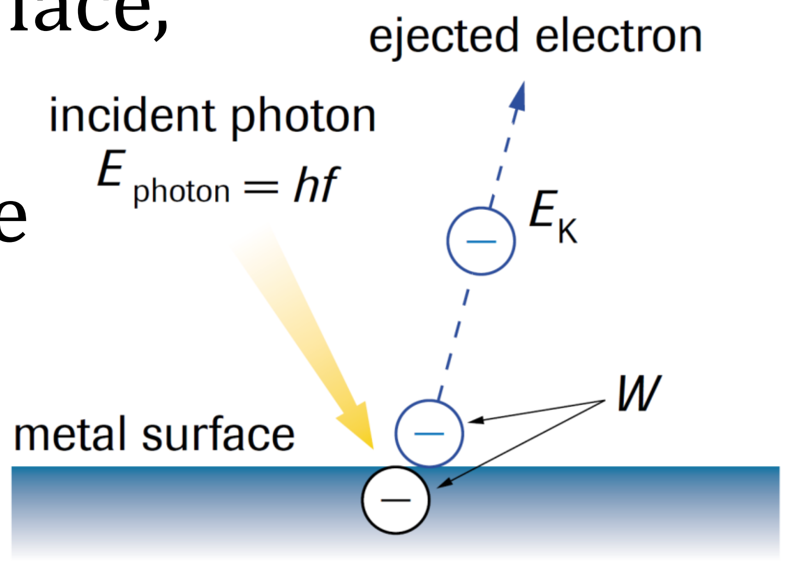
6. During photoemission, the release of the electron is immediate, with no appreciable delay between illumination and photoelectron emission, even for extremely weak light. It appears that the electron absorbs the light energy immediately: no time is required to accumulate sufficient energy to liberate the electrons.

EINSTEIN AND THE PHOTOELECTRIC EFFECT – CONT.

- **Photon:** the quantum of electromagnetic

$$E_{\text{photon}} = hf$$

- When a photon hits a photoelectric surface, a surface electron absorbs its energy
- Some of the energy is needed to release the electron, while the remainder becomes the kinetic energy of the ejected photoelectron



EINSTEIN AND THE PHOTOELECTRIC EFFECT – CONT.

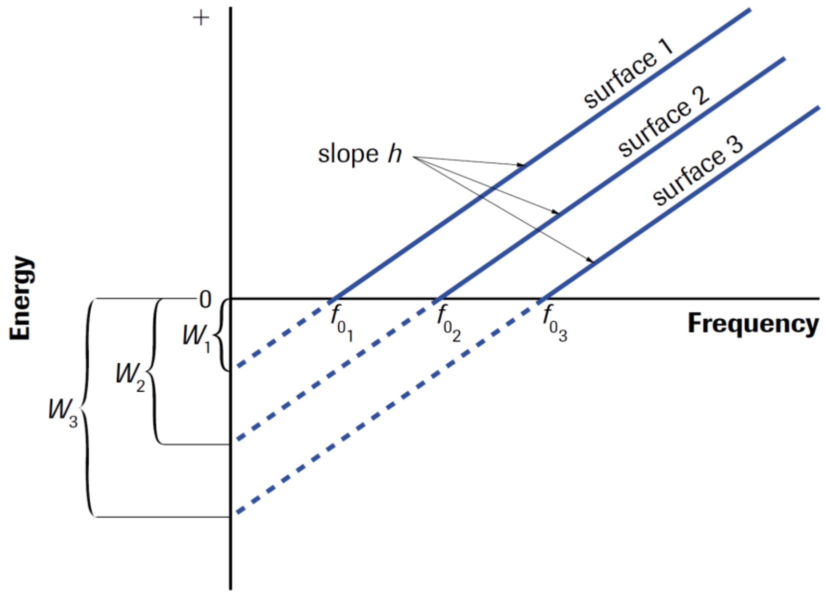
- Mathematically,

$$E_{\text{photon}} = W + E_K$$

- Einstein's Photoelectric Equation

$$\boldsymbol{E_K = hf - \mathbf{W}}$$

- **Work Function (W):** the energy with which an electron is bound to a photoelectric surface



PROBLEM 2

Orange light with a wavelength of 6.00×10^2 nm is directed at a metallic surface with a work function of 1.60 eV. Calculate

(a) the maximum kinetic energy, in joules, of the emitted electrons

(b) their maximum speed

(c) the cutoff potential necessary to stop these electrons

PROBLEM 2 – SOLUTIONS

(a) $\lambda = 6.00 \times 10^2 \text{ nm} = 6.00 \times 10^{-7} \text{ m}$

$h = 6.63 \times 10^{-34} \text{ J·s}$

$E_K = ?$

$$W = 1.60 \text{ eV}$$

$$= (1.60 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})$$

$$W = 2.56 \times 10^{-19} \text{ J}$$

$$E_{K,max} = \frac{hc}{\lambda} - W$$

$$= \frac{(6.63 \times 10^{-34} \text{ J·s})(3.00 \times 10^8 \text{ m/s})}{6.00 \times 10^{-7} \text{ m}} - 2.56 \times 10^{-19} \text{ J}$$

$$E_{K,max} = 7.55 \times 10^{-20} \text{ J}$$

The maximum kinetic energy of the emitted photons is 7.55×10^{-20} J.

PROBLEM 2 – SOLUTIONS

(b) $v = ?$

$m_e = 9.11 \times 10^{-31}$ kg (from Appendix C)

$$E_K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_K}{m}}$$

$$= \sqrt{\frac{2(7.55 \times 10^{-20}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$v = 4.07 \times 10^5\ \text{m/s}$$

The maximum speed of the emitted electrons is 4.07×10^5 m/s.

PROBLEM 2 – SOLUTIONS

(c) $V_0 = ?$

$$E_K = eV_0$$

$$V_0 = \frac{E_K}{e}$$

$$= \frac{7.55 \times 10^{-20}\ \text{J}}{1.60 \times 10^{-19}\ \text{C}}$$

$$V_0 = 0.472\ \text{V}$$

The cutoff potential necessary to stop these electrons is 0.472 V.

The answer to (c) could also have been determined from (a), as follows:

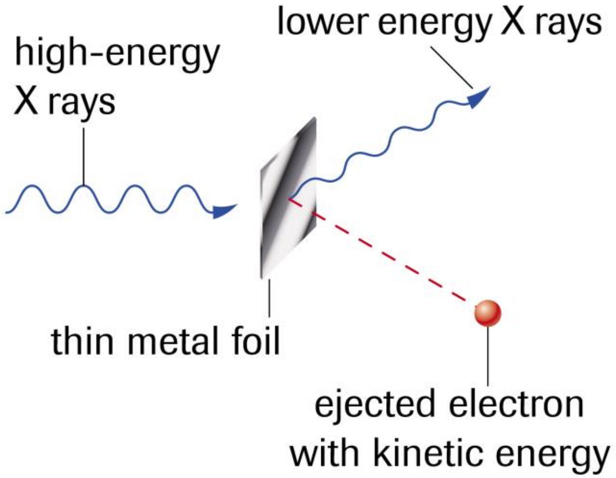
$$7.55 \times 10^{-20}\ \text{J} = \frac{7.55 \times 10^{-20}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = 0.472\ \text{eV}$$

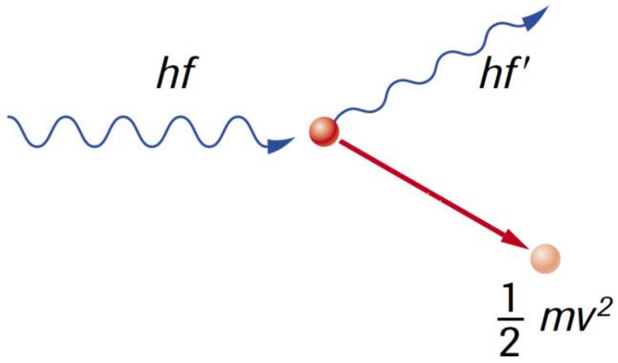
MOMENTUM OF A PHOTON: THE COMPTON EFFECT

- **Compton Effect:** the scattering of photons by high-energy photons
- By combining Einstein's equation from special relativity, the energy of a photon, and the universal wave equation, Compton found the <u>momentum of a photon</u> to be

$$p = \frac{h}{\lambda}$$

  - p – momentum of a photon [kg m/s]
  - h – Planck's constant (6.63×10^{-34} J s)
  - λ – wavelength of the electromagnetic radiation [m]





PROBLEM 3

What is the magnitude of the momentum of a photon with a wavelength of 1.2×10^{-12} m?

PROBLEM 3 – SOLUTIONS

$\lambda = 1.2 \times 10^{-12}$ m

$h = 6.63 \times 10^{-34}$ J·s

$p = ?$

$$p = \frac{h}{\lambda}$$

$$= \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{1.2 \times 10^{-12}\ \text{m}}$$

$$p = 5.5 \times 10^{-22}\ \text{kg}\cdot\text{m/s}$$

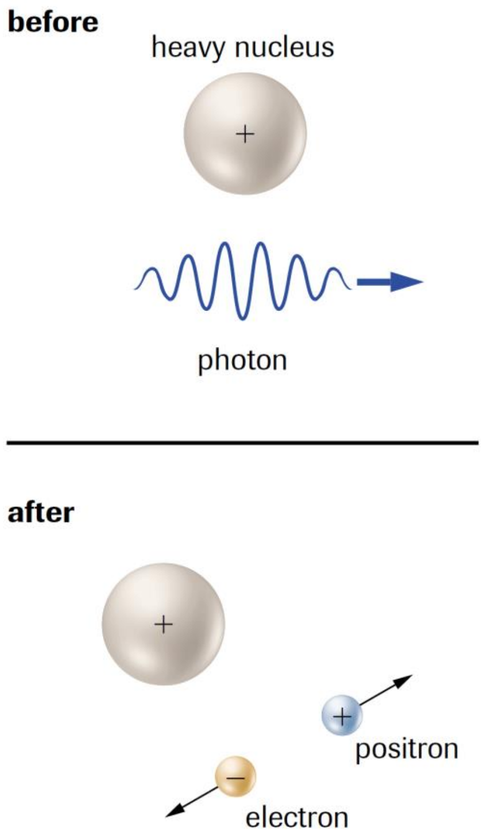
The magnitude of the momentum of the photon is 5.5×10^{-22} kg·m/s.

INTERACTIONS OF PHOTONS WITH MATTER

1. The most common interaction is simple reflection, as when photons of visible light undergo perfectly elastic collisions with a mirror.
2. In the case of the photoelectric effect, a photon may liberate an electron, being absorbed in the process.
3. In the Compton effect, the photon emerges with less energy and momentum, having ejected a photoelectron. After its interaction with matter, the photon still travels at the speed of light but is less energetic, having a lower frequency.
4. A photon may interact with an individual atom, elevating an electron to a higher energy level within the atom. In this case, the photon completely disappears. All of its energy is transferred to the atom, causing the atom to be in an energized, or “excited,” state. (We will examine the details of this interaction later in this chapter.)

INTERACTIONS OF PHOTONS WITH MATTER

5. A photon can disappear altogether, creating two particles of nonzero mass in a process called pair production.
   - **Pair Production:** the creation of a pair of particles (an electron and a positron) as a result of a collision of a high-energy photon with a nucleus
   - Pair production requires a photon of very high energy (1.02 MeV) and, correspondingly, a very short wavelength (as with X-ray and gamma-ray photons).
     - When such a photon collides with a heavy nucleus, it disappears, creating an electron (${}^{-}_{0}e$) and a particle of equal mass but opposite charge, the positron (${}^{+}_{0}e$). This creation of mass from energy obeys Einstein's mass–energy equivalence equation $E = mc^2$.



SUMMARY – FOUNDATIONS OF QUANTUM THEORY

- A blackbody of a given temperature emits electromagnetic radiation over a continuous spectrum of frequencies, with a definite intensity maximum at one particular frequency. As the temperature increases, the intensity maximum shifts to progressively higher frequencies.
- Planck proposed that molecules or atoms of a radiating blackbody are constrained to vibrate at discrete energy levels, which he called quanta. The energy of a single quantum is directly proportional to the frequency of the emitted radiation, according to the relationship $E = hf$, where h is Planck's constant.
- Photoelectrons are ejected from a photoelectric surface when the incident light is above a certain frequency f_0, called the threshold frequency. The intensity (brightness) of the incoming light has no effect on the threshold frequency. The threshold frequency is different for different surfaces.
- The cutoff potential is the potential difference at which even the most energetic photoelectrons are prevented from reaching the anode. For the same surface the cutoff potential is different for each frequency, and the higher the frequency of the light, the higher the cutoff potential.
- The energy of light is transmitted in bundles of energy called photons, whose energy has a discrete, fixed amount, determined by Planck's equation, $E = hf$.
- When a photon hits a photoelectric surface, a surface electron absorbs its energy. Some of the absorbed energy releases the electron, and the remainder becomes its kinetic energy of the liberated electron, according to the photoelectric equation $E_K = hf - W$.
- In the Compton effect, high-energy photons strike a surface, ejecting electrons with kinetic energy and lower-energy photons. Photons have momentum whose magnitude is given by $p = \frac{\lambda}{h}$.
- Interactions between photons and matter can be classified into reflection, the photoelectric effect, the Compton effect, changes in electron energy levels within atoms, and pair production.

PRACTICE

Readings

- Section 12.1 (pg 594)

Questions

- pg 608 #2,10,12,15